Extracting Dynamics from
Ocean Wave Time Series Data[1]

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Abstract

Recent developments in chaotic dynamics have indicated that qualitative information of a dynamical system can be extracted from the observation of a single time series as the time series bears the marks of all other variables relevant in the underlying dynamics. In this paper we review this approach and explore its possible applications to ocean wave dynamics.

Introduction

The study of ocean wave dynamics has been done generally using the equations of motion. The difficulty in verifying the theoretical analysis with measurements lies in the specification of an initial state that requires the measurement of functions over a three-dimensional domain. Acquisition of such measurements is usually prohibitive. Typical experiments employ wave probes that produce time series of surface fluctuations only. A time series of one variable generally provides a limited amount of dynamical information. Recent developments in chaotic dynamics have advanced theorems that furnish a procedure for reconstructing a dynamical system from the observation of a single variable. Thus, a time series bears the marks of all other variables relevant in the underlying dynamics, and key features of the dynamics can be extracted from a given time series. In this paper we expect to explore the chaotic dynamics approach and its application to the study of ocean wave dynamics. Specifically we examine if it is possible to identify an attractor for an ocean wave time series and determine its dimensionality as well as the minimal

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dimensionality of the phase space within which the attractor is embedded.

Dynamic Systems and Phase Space

The study of chaotic dynamics provides a new and stimulating approach to the study of fluid flow. The basic proposition is that relatively simple systems of coupled nonlinear first-order equations often have chaotic solutions. These solutions -- sometimes called strange attractors -- are much more irregular than solutions of deterministic equations. This has generated the hypothesis that some of the fluid flow problems can be qualitatively explained by models that are highly simplified in comparison with full hydrodynamic equations. While the dynamic systems approach has not yet achieved much quantitative predictive power at the present, it has provided a significant new direction of study.

Following Lorenz (1963) we consider a dynamic system formally as:

$$dy/dt = F(y), \tag{1}$$

where time t is the single independent varable, $y = (y_1, y_2, \ldots y_n)$ represents a state of the system and may be thought of as a point in a suitably defined space -- usually called phase space, and the vector field $F(y)$ is in general a non-linear operator acting on points in the phase space. A state which is varying in accordance with (1) is represented by a moving particle traveling along a trajectory in phase space. The trajectory becomes a strange attractor when it is chaotic, sensitively depends on the initial conditions, and it is attracted to a bounded region in phase space.

Many current studies on chaotic dynamics have focused on understanding and characterization of strange attractors. Strange attractors can be generally characterized through quantities like Kolmogorov entropy, Lyapunov exponents, and generalized dimensions. If the governing equations are known, then there are reliable methods for determining these quantities. If however only measurements of time series are available, then the problem becomes much more difficult. In this paper we expect to pursue this latter course without immediately resorting to our established knowledge of ocean wave dynamics. In particular, we are primarily interested in the applicability of the various approaches of characterizing strange attractors and determining basic degrees of freedom of the system that govern the quantitative predictability of the dynamic system.

Kolmogorov Entropy

Due to the sensitivity of the chaotic dynamic system to the initial conditions, trajectories arising from two different but indistinguishable initial conditions, within a given precision, will evolve into distinct states after a finite interval of time. Thus chaotic trajectories reveal new informations about the system continuously. The Kolmogorov entropy (Komogorov, 1959) has been used to measure the rate at which information is being created by the dynamical system. It is zero for non-chaotic and infinite for random systems. A finite and positive Komogorov entropy usually implies deterministic chaos. Detailed studies of Komogorov entropy were given in Farmer et al. (1983), Grassbeger and Procaccia (1983) and others.

Lyapunov Exponents

Lyapunov exponent quantifies the local dynamical behavior or the average stability of trajectories on an attractor that are determined by the response of the system to small pertubations. Positive Lyapunov exponents generally indicate orbital divergence of the trajectory and hence chaotic motion. For non-chaotic systems all exponents are negative or zero. Algorithms have been developed to extract the Lyapunov exponents of an attractor reconstructed from a measured time series, e.g., Wolf et al. (1985), Eckmann and Ruelle (1985), and recently Bryant et al. (1990). A comparison of the various earlier algorithms for determining Lyapunov exponents from experimental data is given in Vastano and Kostelich (1986).

Generalized Dimensions

Dimension is one of the most basic properties of geometric objects. Basically the dimension of a space is the amount of information needed to specify points in the space accurately. For dynamics the dimension provides an indication of the number of essential variables required to represent the dynamics. The dimensionality of a phase space, since it controls the number of possible states, will therefore be associated with the number of a priori degrees of freedom of the system. There are many possible characterizations of dimension, among them the correlation dimension introduced by Grassberger and Procaccia (1983) has become the most widely used approach for estimating generalized dimension from time series data. A detailed review and analysis of this method is given by Theiler (1988).

Reconstruction of an Attractor

Perhaps one the most interesting and enticing results developed from the chaotic dynamics is the notion that it is possible to reconstruct certain properties of an attractor in phase space from the time series of a single variable. Following the earlier works of Packard et al. (1980) and Takens (1981), the basic principle is to create a set of m-dimensional vectors from a single time series $x_i = x(t_i)$, $i = 1,\ldots,N$, with the x_i corresponding to measurements in time. This process is known as 'embedding' and m is the 'embedding dimension'. The reconstruction is accomplished by introducing a time lag p such that the m-dimensional vectors have the form

$$X_i = [x(t_i),\ x(t_i + p), \ldots,\ x(t_i + (m - 1)p)]. \quad (2)$$

In principle, the various characterizations -- the Komogorov entropy, the Lyapunov exponents, and the generalized dimensions -- are all accessible through this reconstruction (Simm et al., 1987).

Concluding Remarks

With this very brief note serving as a review and summary, we have outlined the various aspects we expect to pursue in order to explore the chaotic dynamics approach and its application to the ocean waves. Detailed analyses of measured ocean wave time series will be presented at the conference.

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STABILITY OF NEARLY BREAKING LONG WAVES

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Abstract

This paper addresses the development of equations for nonlinear dispersive waves that have improved stability properties over the commonly used Boussinesq-type equations. Boussinesq-type equations are unstable in the short-wave regime, which, although outside the range of applicability of the Boussinesq equations, do appear in the numerical solution of long wave problems as the latter approach the shoreline and begin to break. The basic hypothesis for the development of stable equations is that the Hamiltonian corresponding to Boussinesq-type equations either does not exist at all or becomes negative in the presence of very short waves. This is due to the insufficient approximation of the kinetic energy of the flow, which in the presence of short waves, fails to remain positive. The present work seeks approximations of the kinetic energy that remain positive regardless of wave length.

Introduction

The mathematical foundation for nonlinear dispersive waves is provided by the theory of Boussinesq, which allows moderate curvature of the free surface. Certain terms appear in the governing equations that account for the effects of wave dispersion. These terms are typically of order $\mu = (h/L)^2$, where h is the water depth and L a typical wave length. For very long waves the $O(\mu)$ terms are negligible, thus allowing the ordinary shallow-water equations to be recovered. In theory, the Boussinesq equations are limited to finite but small wave amplitude, due to the fact that their derivation is based on an expansion of the flow quantities in a power series of a small parameter, $\epsilon = a/h$, where a is a measure of the wave amplitude. Subsequently, the equations are

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